Journal of Guidance, Control, and Dynamics
Vol. 31, No. 2, March–April 2008

Erratum

Influence of Wind Speed on Airship Dynamics

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[***Journal of Guidance, Control, and Dynamics*, 25, (6), pp. 1116–1124, 2002**]

DOI: 10.2514/1.33409

I. Introduction

IN THIS Erratum the authors revise the paper "Influence of Wind Speed on Airship Dynamics" [1]. After a careful mathematical deduction, it was verified that this influence under a *constant translation wind* indeed does not exist, and the results presented by Thomasson [2,3] are confirmed. In other words, the airship dynamics equation is the same when expressed in terms of the inertial velocity with no wind or in terms of the relative air velocity under a constant translation wind.

To confirm the results, the airship dynamics is expressed using the Lagrangian approach, instead of the original Newtonian method used in Azinheira et al. [1]. In addition, the new kinematics representation with quaternions used in Azinheira et al. [4], describing the velocity changes from local to Earth frames, is used in the derivation of a compact form of the force equation. Assuming a constant translation wind, the new force equation written in terms of inertial and wind velocities is then transformed to be a function of the relative velocities. The force equation expressed in this way shows the same structure and the same terms as the one expressed with the inertial and wind velocities, confirming the invariance of the dynamics under a constant translation wind.

In a last step, some terms in the kinematics force equation are discarded, as they are already accounted for in the aerodynamic forces. These terms are proportional to the squared relative velocity, and their influence is already considered through the aerodynamic coefficients measured in wind-tunnel experiments. This final step is particularly important for simulation purposes, otherwise some terms would be considered twice in the dynamic model. Simulation results, obtained using the AURORA simulator platform support the theoretical conclusions.

II. Derivation of the Torque Equation

Considering the angular momentum given in Eq. (18) of the original work [1], derived from the three components consisting of the angular momentum of the vehicle itself, plus a term corresponding to the buoyancy air, plus a virtual term proportional to the relative angular speed, we have

$$H = J_a\omega + OC \times mV - J_{Ba}\omega_w \tag{1}$$

where J_a is the apparent inertia matrix of the airship, and J_{Ba} is the apparent inertia matrix of the buoyancy air, both referenced to the origin of the local frame at the center of volume (CV). Also, OC is the vector from the CV to the center of gravity (CG), V is the airship linear velocity in local frame, ω is its angular velocity, and ω_w is the wind angular velocity.

The torque equation (9) in Azinheira et al. [1] was obtained through the derivation of the angular momentum (1) as

$$\begin{aligned} T = \dot{H} + \omega \times H &= (J_a\dot{\omega} + \omega \times J_a\omega) \\ &\quad + [OC \times m\dot{V} + \omega \times (OC \times mV)] \\ &\quad - (J_{Ba}\dot{\omega}_w + \omega \times J_{Ba}\omega_w) \end{aligned} \tag{2}$$

or, in matrix form:

$$\begin{aligned} [mOC \times J_a]\dot{x} = T + T_{kw} &= T - \omega \times J_a\omega - \omega \times (OC \times mV) \\ &\quad + J_{Ba}\dot{\omega}_w + \omega \times J_{Ba}\omega_w \end{aligned}$$

From this equation, and following Eqs. (21) and (22) from Azinheira et al. [1], the contribution of wind and kinematics was pointed out as

$$T_{kw} = -\omega \times J_a\omega - \omega \times (OC \times mV) + J_{Ba}\dot{\omega}_w + \omega \times J_{Ba}\omega_w \tag{3}$$

However, as the reference considered is the origin of the local frame CV, and not CG, the correct differentiation of the torque equation (2) should include also the translation of the angular momentum vector parallel to itself, yielding a different result. To confirm this, the airship dynamics is expressed using the Lagrangian approach, instead of the original Newtonian method used in Azinheira et al. [1]. The complete development will be omitted here, but the final results prove to be in complete agreement with the dynamics under a moving fluid presented by Thomasson [2,3].

As a first step, let us recall from Azinheira et al. [1] the three components of the total kinetic energy of the airship:

1) The energy of the vehicle itself, $W_c = \frac{1}{2}x_c^T\overline{M_c}x_c$, considered as a rigid body with CG located at a distance OC from the center of reference CV, 6-D inertial velocity x_c referenced to the CG, and 6-D inertial matrix

$$\overline{M_c} = \begin{bmatrix} mI & 0 \\ 0 & J_c \end{bmatrix}$$

2) The energy added to the buoyancy volume, $W_B = -\frac{1}{2}x^T\overline{M_B}x + \frac{1}{2}x_a^T\overline{M_B}x_a$, where x indicates the 6-D inertial velocity referenced to the CV, and x_a indicates the airship relative velocity given by $x_a = x - x_w$, with wind velocity x_w. $\overline{M_B}$ is the 6-D inertial mass matrix of the buoyancy air.

3) The energy of the virtual mass, $W_v = \frac{1}{2}x_a^T\overline{M_v}x_a$, where $\overline{M_v}$ is the 6-D generalized virtual mass matrix.

For the first term, the change of the velocity vector from CG to CV is given by $V_c = V - OC \times \omega$ or

$$x_c = \begin{bmatrix} I & -C_3 \\ 0 & I \end{bmatrix}$$

where the synthetic matrix notation for the cross product $C_3 = OC\times$ is used. This leads to

$$W_c = \frac{1}{2} x^T \begin{bmatrix} I & 0 \\ C_3 & I \end{bmatrix} \overline{M_c} \begin{bmatrix} I & -C_3 \\ 0 & I \end{bmatrix} \qquad x = \frac{1}{2} x^T \overline{M_O} x$$

where

$$\overline{M_O} = \begin{bmatrix} mI & -mC_3 \\ mC_3 & J_O \end{bmatrix}$$

with $J_O = J_c - mC_3^2$.

If the relative velocity is expressed as a function of the wind and inertial velocities, then, the total kinetic energy can be written as

$$\begin{aligned} W &= W_c + W_v + W_B \\ &= \tfrac{1}{2} x^T \overline{M_O} x + \tfrac{1}{2} x^T \overline{M_v} x + \tfrac{1}{2} x_w^T \overline{M_v} x_w - x^T \overline{M_v} x_w \\ &\quad + \tfrac{1}{2} x_w^T \overline{M_B} x_w - x^T \overline{M_B} x_w \\ &= \tfrac{1}{2} x^T \overline{M_a} x - x^T \overline{M_{Ba}} x_w + \tfrac{1}{2} x_w^T \overline{M_{Ba}} x_w \end{aligned}$$

where

$$\overline{M_a} = \overline{M_O} + \overline{M_v} = \begin{bmatrix} M_a & -mC_3 \\ mC_3 & J_a \end{bmatrix}$$

is the 6-D apparent mass matrix, and

$$\overline{M_{Ba}} = \overline{M_B} + \overline{M_v} = \begin{bmatrix} m_B I + M_v & 0 \\ 0 & J_B + J_v \end{bmatrix}$$

is the 6-D apparent mass matrix of the buoyancy air.

The Lagrangian equations of motion may be given by

$$F(\dot{q}, q) = \frac{d}{dt} \frac{\partial W}{\partial \dot{q}} - \frac{\partial W}{\partial q} \tag{4}$$

where $W(q, \dot{q})$ is the system kinetic energy expressed as a function of the generalized coordinates q and their time derivatives $\dot{q}$, and $F(\dot{q}, q) = F_q$ is the generalized force vector. The generalized position vector is defined as $q = [p^T, e^T]$, where p and e are the position and quaternion vectors, respectively.

The development of the Lagrangian equation (4), followed by the transformation $F_x = T^T F_q$ (see Azinheira et al. [4] for used notation) yields the correct dynamic equation for the torque:

$$\begin{aligned} [mOC \times J_a]\dot{x} &= T + T_{kw} = T - \omega \times J_a \omega - \omega \times (OC \times mV) \\ &\quad + J_{Ba} \dot{\omega}_w + \omega \times J_{Ba} \omega_w + mV \times (OC \times \omega) - V_a \times M_{Ba} V_a \end{aligned} \tag{5}$$

Thus, the kinematics/wind torque contribution is, in fact, given by

$$\begin{aligned} T_{kw} &= -\omega \times J_a \omega - \omega \times (OC \times mV) + J_{Ba} \dot{\omega}_w + \omega \times J_{Ba} \omega_w \\ &\quad + mV \times (OC \times \omega) - V_a \times M_{Ba} V_a \end{aligned} \tag{6}$$

where two new terms appear regarding the old formulation (3).

It can also be shown that the dynamic equation is invariant through a steady translation wind. Developing Eq. (5) in terms of the relative air velocity $x_a = x - x_w$, and considering $\ddot{p}_w = 0 \Rightarrow \dot{V}_w = -\omega \times V_w$, and $\omega_w = 0$ yields

$$\begin{aligned} [mOC \times J_a]\dot{x}_a &= T + T_{kw} = -\omega \times J_a \omega - \omega \times (OC \times mV_a) \\ &\quad + mV_a \times (OC \times \omega) - V_a \times M_{Ba} V_a \end{aligned} \tag{7}$$

which corresponds to the torque (5) expressed in the inertial frame with no wind, therefore confirming the results presented by Thomasson [2].

It is important to note that the last term in Eq. (6) is already considered in the aerodynamic forces. In fact, this term is proportional to the squared airspeed and is therefore included in the dynamic model through the aerodynamic coefficients obtained in wind-tunnel experiments. This leaves the final kinematics/wind torque equation as

$$\begin{aligned} T_{kw} &= -\omega \times J_a \omega - \omega \times (OC \times mV) \\ &\quad + J_{Ba} \dot{\omega}_w + \omega \times J_{Ba} \omega_w + mV \times (OC \times \omega) \end{aligned} \tag{8}$$

The difference, when compared to the previous formulation (3), is indeed only the last term added.

In order to verify that the dynamics resulting for the AURORA model near air hovering is unchanged by a constant translation wind input, the correction is introduced in the model. The results from the simulator show that the poles effectively present no influence from wind despite the wind terms in Eq. (5).

The second test on the dynamic model is to verify the evolution of the longitudinal and lateral poles with varying airspeed, from air hover to aerodynamic flight. The results, as expected, show negligible influence from the wind.

III. Conclusions

The development presented here corrects a formulation error in the original paper. After a careful deduction using the Lagrangian approach, the rectification of the dynamic model resulted in a new term added to the previous dynamic equation.

As a consequence, the corrected airship dynamics is shown to be, indeed, independent of the wind input. In other words, the dynamic equation is the same when expressed in terms of the inertial velocity with no wind or in terms of the air velocity under a constant translation wind. The model simulation, introducing this single term, confirms that the airship dynamics is invariant under a steady translation wind, for the entire flight envelope.

Acknowledgment

We would like to thank P. G. Thomasson for pointing out the error in the original formulation and also for his revisions and suggestions during the new developments on the airship modeling.

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